4.1 Writing and Graphing Inequalities

Essential Question How can you use a number line to represent solutions of an inequality?

1 ACTIVITY: Understanding Inequality Statements

Work with a partner. Read the statement. Circle each number that makes the statement true, and then answer the questions.

a. "You are in at least 5 of the photos."

−3 −2 −1 0 1 2 3 4 5 6

- What do you notice about the numbers that you circled?
- Is the number 5 included? Why or why not?
- Write four other numbers that make the statement true.

b. "The temperature is less than −4 degrees Fahrenheit."

−7 −6 −5 −4 −3 −2 −1 0 1 2

- What do you notice about the numbers that you circled?
- Can the temperature be exactly −4 degrees Fahrenheit? Explain.
- Write four other numbers that make the statement true.

c. "More than 3 students from our school are in the chess tournament."

−3 −2 −1 0 1 2 3 4 5 6

- What do you notice about the numbers that you circled?
- Is the number 3 included? Why or why not?
- Write four other numbers that make the statement true.

d. "The balance in a yearbook fund is no more than −$5."

−7 −6 −5 −4 −3 −2 −1 0 1 2

- What do you notice about the numbers that you circled?
- Is the number −5 included? Why or why not?
- Write four other numbers that make the statement true.

Common Core

Inequalities

In this lesson, you will

- write and graph inequalities.
- use substitution to check whether a number is a solution of an inequality.

Preparing for Standard 7.EE.4b

2 ACTIVITY: Understanding Inequality Symbols

Work with a partner.

a. Consider the statement "x is a number such that $x > -1.5$."

- Can the number be exactly -1.5? Explain.
- Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
- Write four other numbers that are not integers that make the statement true.

b. Consider the statement "x is a number such that $x \leq \frac{5}{2}$."

- Can the number be exactly $\frac{5}{2}$? Explain.
- Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
- Write four other numbers that are not integers that make the statement true.

3 ACTIVITY: Writing and Graphing Inequalities

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of x that make the inequality true.

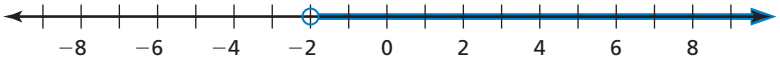
Math Practice 1

Check Progress

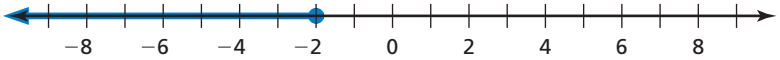
All the graphs are similar. So, what can you do to make sure that you have correctly written each inequality?

a.

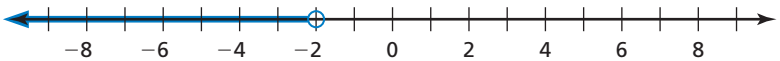
b.



c.



d.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to represent solutions of an inequality?

5. **STRUCTURE** Is $x \geq -1.4$ the same as $-1.4 \leq x$? Explain.

Practice Use what you learned about writing and graphing inequalities to complete Exercises 4 and 5 on page 128.

4.1 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
inequality, *p. 126*
solution of an inequality, *p. 126*
solution set, *p. 126*
graph of an inequality, *p. 127*

An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine where to place the inequality symbol.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 Writing an Inequality

A number q plus 5 is greater than or equal to -7.9. Write this word sentence as an inequality.

A number q plus 5 is greater than or equal to -7.9.

$q + 5$ $\geq$ -7.9

∴ An inequality is $q + 5 \geq -7.9$.

On Your Own

Now You're Ready
Exercises 6–9

Write the word sentence as an inequality.

1. A number x is at most -10.
2. Twice a number y is more than $-\frac{5}{2}$.

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 2 \leq -1$	Is the inequality true?
-2	$-2 + 2 \stackrel{?}{\leq} -1$ $0 \not\leq -1$ ✗	no
-3	$-3 + 2 \stackrel{?}{\leq} -1$ $-1 \leq -1$ ✓	yes
-4	$-4 + 2 \stackrel{?}{\leq} -1$ $-2 \leq -1$ ✓	yes

Reading
The symbol $\not\leq$ means *is not less than or equal to*.

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Checking Solutions

Tell whether -2 is a solution of each inequality.

a. $y - 5 \geq -6$

$y - 5 \geq -6$ Write the inequality.

$-2 - 5 \stackrel{?}{\geq} -6$ Substitute -2 for y.

$-7 \not\geq -6$ ✗ Simplify.

-7 is *not* greater than or equal to -6.

∴ So, -2 is *not* a solution of the inequality.

b. $-5.5y < 14$

$-5.5y < 14$

$-5.5(-2) \stackrel{?}{<} 14$

$11 < 14$ ✓

11 is less than 14.

∴ So, -2 is a solution of the inequality.

On Your Own

Now You're Ready
Exercises 11–16

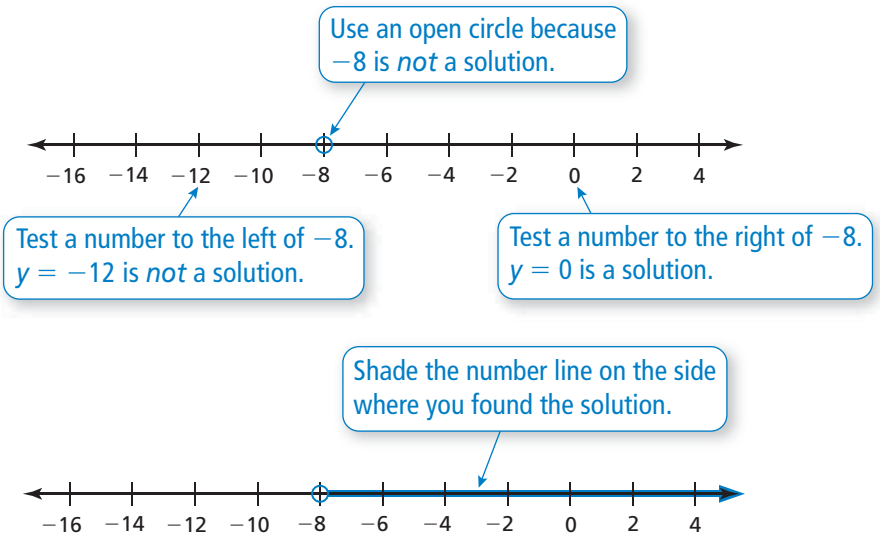
Tell whether -5 is a solution of the inequality.

3. $x + 12 > 7$ **4.** $1 - 2p \leq -9$ **5.** $n \div 2.5 \geq -3$

The **graph of an inequality** shows all the solutions of the inequality on a number line. An open circle ○ is used when a number is *not* a solution. A closed circle ● is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 Graphing an Inequality

Graph $y > -8$.



Study Tip

The graph in Example 3 shows that the inequality has *infinitely many* solutions.

On Your Own

Now You're Ready
Exercises 17–20

Graph the inequality on a number line.

6. $x < -1$ **7.** $z \geq 4$ **8.** $s \leq 1.4$ **9.** $-\frac{1}{2} < t$

4.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

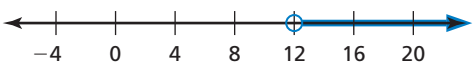
1. **PRECISION** Should you use an open circle or a closed circle in the graph of the inequality $b \geq -42$? Explain.
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

k is less than or equal to -3.	k is no more than -3.
k is at most -3.	k is at least -3.

3. **REASONING** Do $x < 5$ and $5 < x$ represent the same inequality? Explain.

Practice and Problem Solving

Write an inequality for the graph. Then, in words, describe all the values of x that make the inequality true.

4. 

5.

Write the word sentence as an inequality.

1 6. A number y is no more than -8.

7. A number w added to 2.3 is more than 18.

8. A number t multiplied by -4 is at least $-\frac{2}{5}$.

9. A number b minus 4.2 is less than -7.5.

10. **ERROR ANALYSIS** Describe and correct the error in writing the word sentence as an inequality.

> ✗ Twice a number x is at most −24.
>
> $2x \geq -24$

Tell whether the given value is a solution of the inequality.

2 11. $n + 8 \leq 13; n = 4$

12. $5h > -15; h = -5$

13. $p + 1.4 \leq 0.5; p = 0.1$

14. $\frac{a}{6} > -4; a = -18$

15. $-\frac{2}{3}s \geq 6; s = -9$

16. $\frac{7}{8} - 3k < -\frac{1}{2}; k = \frac{1}{4}$

Graph the inequality on a number line.

3 17. $r \leq -9$

18. $g > 2.75$

19. $x \geq -3\frac{1}{2}$

20. $z < 1\frac{1}{4}$

21. **FOOD TRUCK** Each day at lunchtime, at least 53 people buy food from a food truck. Write an inequality that represents this situation.

Tell whether the given value is a solution of the inequality.

22. $4k < k + 8; k = 3$

23. $\frac{w}{3} \geq w - 12; w = 15$

24. $7 - 2y > 3y + 13; y = -1$

25. $\frac{3}{4}b - 2 \leq 2b + 8; b = -4$

26. **MODELING** A subway ride for a student costs \$1.25. A monthly pass costs \$35.

a. Write an inequality that represents the number of times you must ride the subway for the monthly pass to be a better deal.

b. You ride the subway about 45 times per month. Should you buy the monthly pass? Explain.

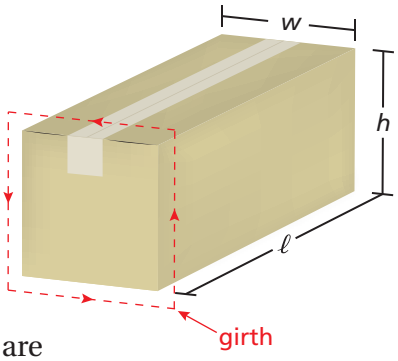
27. **LOGIC** Consider the inequality $b > -2$.

a. Describe the values of b that are solutions of the inequality.

b. Describe the values of b that are *not* solutions of the inequality. Write an inequality for these values.

c. What do all the values in parts (a) and (b) represent? Is this true for any inequality?

28. Critical Thinking A postal service says that a rectangular package can have a maximum combined length and *girth* of 108 inches. The girth of a package is the distance around the perimeter of a face that does not include the length.



a. Write an inequality that represents the allowable dimensions for the package.

b. Find three different sets of allowable dimensions that are reasonable for the package. Find the volume of each package.

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 3.3)*

29. $p - 8 = 3$ **30.** $8.7 + w = 5.1$ **31.** $x - 2 = -9$

32. **MULTIPLE CHOICE** Which expression has a value less than -5? *(Section 1.2)*

(A) $5 + 8$ (B) $-9 + 5$ (C) $1 + (-8)$ (D) $7 + (-2)$

4.2 Solving Inequalities Using Addition or Subtraction

Essential Question How can you use addition or subtraction to solve an inequality?

1 ACTIVITY: Writing an Inequality

Work with a partner. Members of the Boy Scouts must be less than 18 years old. In 4 years, your friend will still be eligible to be a scout.

a. Which of the following represents your friend's situation? What does x represent? Explain your reasoning.

$x + 4 > 18$ $\quad$ $x + 4 < 18$

$x + 4 \geq 18$ $\quad$ $x + 4 \leq 18$

b. Graph the possible ages of your friend on a number line. Explain how you decided what to graph.

2 ACTIVITY: Writing an Inequality

Work with a partner. Supercooling is the process of lowering the temperature of a liquid or a gas below its freezing point without it becoming a solid. Water can be supercooled to 86°F below its normal freezing point (32°F) and still not freeze.

a. Let x represent the temperature of water. Which inequality represents the temperature at which water can be a liquid or a gas? Explain your reasoning.

$x - 32 > -86$ $\quad$ $x - 32 < -86$

$x - 32 \geq -86$ $\quad$ $x - 32 \leq -86$

b. On a number line, graph the possible temperatures at which water can be a liquid or a gas. Explain how you decided what to graph.

Common Core

Inequalities

In this lesson, you will

- solve inequalities using addition or subtraction.
- solve real-life problems.

Learning Standard 7.EE.4b

3 ACTIVITY: Solving Inequalities

Math Practice 4

Interpret Results

What does the solution of the inequality represent?

Work with a partner. Complete the following steps for Activity 1. Then repeat the steps for Activity 2.

- Use your inequality from part (a). Replace the inequality symbol with an equal sign.
- Solve the equation.
- Replace the equal sign with the original inequality symbol.
- Graph this new inequality.
- Compare the graph with your graph in part (b). What do you notice?

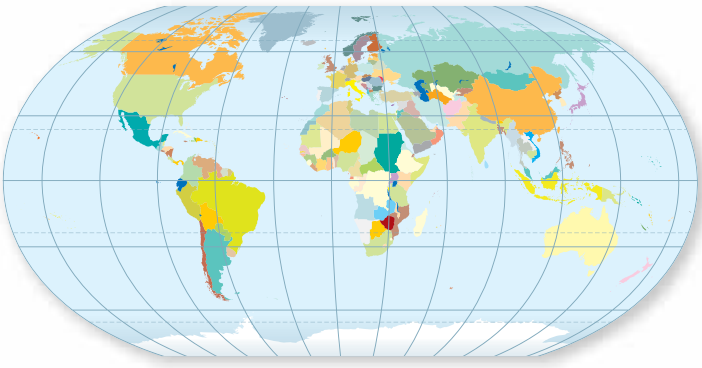
4 ACTIVITY: Temperatures of Continents

Work with a partner. The table shows the lowest recorded temperature on each continent. Write an inequality that represents each statement. Then solve and graph the inequality.

a. The temperature at a weather station in Asia is more than 150°F greater than the record low in Asia.

b. The temperature at a research station in Antarctica is at least 80°F greater than the record low in Antarctica.

Continent	Lowest Temperature
Africa	−11°F
Antarctica	−129°F
Asia	−90°F
Australia	−9.4°F
Europe	−67°F
North America	−81.4°F
South America	−27°F



What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use addition or subtraction to solve an inequality?

6. Describe a real-life situation that you can represent with an inequality. Write the inequality. Graph the solution on a number line.

Practice Use what you learned about solving inequalities to complete Exercises 3–5 on page 134.

4.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Ideas

Study Tip

You can solve inequalities in the same way you solve equations. Use inverse operations to get the variable by itself.

Addition Property of Inequality

Words When you add the same number to each side of an inequality, the inequality remains true.

Numbers

$$\begin{array}{rcl} -4 & < & 3 \\ \underline{+\,2} & & \underline{+\,2} \\ -2 & < & 5 \end{array}$$

Algebra If $a < b$, then $a + c < b + c$.

If $a > b$, then $a + c > b + c$.

Subtraction Property of Inequality

Words When you subtract the same number from each side of an inequality, the inequality remains true.

Numbers

$$\begin{array}{rcl} -2 & < & 2 \\ \underline{-\,3} & & \underline{-\,3} \\ -5 & < & -1 \end{array}$$

Algebra If $a < b$, then $a - c < b - c$.

If $a > b$, then $a - c > b - c$.

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 Solving an Inequality Using Addition

Solve $x - 5 < -3$. Graph the solution.

$x - 5 < -3$ Write the inequality.

Undo the subtraction. → $\underline{+\,5} \quad \underline{+\,5}$ Addition Property of Inequality

$x < 2$ Simplify.

∴ The solution is $x < 2$.

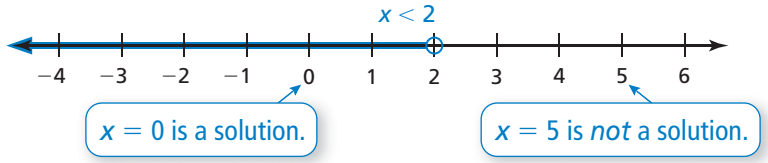
Check:

$x = 0$: $0 - 5 \stackrel{?}{<} -3$

$-5 < -3$ ✓

$x = 5$: $5 - 5 \stackrel{?}{<} -3$

$0 \not< -3$ ✗



On Your Own

Solve the inequality. Graph the solution.

1. $y - 6 > -7$
2. $b - 3.8 \leq 1.7$
3. $-\frac{1}{2} > z - \frac{1}{4}$

EXAMPLE 2 Solving an Inequality Using Subtraction

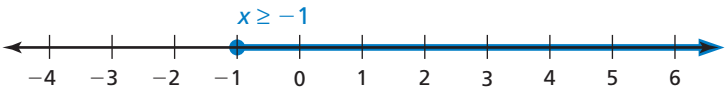
Solve $13 \le x + 14$. Graph the solution.

$13 \le x + 14$ Write the inequality.

Undo the addition. → $-14 \quad -14$ Subtraction Property of Inequality

$-1 \le x$ Simplify.

∴ The solution is $x \ge -1$.



Reading

The inequality $-1 \le x$ is the same as $x \ge -1$.

On Your Own

Now You're Ready
Exercises 3–17

Solve the inequality. Graph the solution.

4. $w - 7 \le -10$ **5.** $-7.5 \ge d - 10$ **6.** $x + \frac{3}{4} > 1\frac{1}{2}$

EXAMPLE 3 Real-Life Application

A person can be no taller than 6.25 feet to become an astronaut pilot for NASA. Your friend is 5 feet 9 inches tall. Write and solve an inequality that represents how much your friend can grow and still meet the requirement.

Words Current height | plus | amount your friend can grow | is no more than | the height limit.

Variable Let h be the possible amounts your friend can grow.

Inequality $5.75 + h \le 6.25$

5 ft 9 in. $= 60 + 9 = 69$ in.

$69 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 5.75 \text{ ft}$

$5.75 + h \le 6.25$ Write the inequality.

$-5.75 \quad -5.75$ Subtraction Property of Inequality

$h \le 0.5$ Simplify.

∴ So, your friend can grow no more than 0.5 foot, or 6 inches.

On Your Own

7. Your cousin is 5 feet 3 inches tall. Write and solve an inequality that represents how much your cousin can grow and still meet the requirement.

4.2 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **REASONING** Is the inequality $c + 3 > 5$ the same as $c > 5 - 3$? Explain.

2. **WHICH ONE DOESN'T BELONG?** Which inequality does *not* belong with the other three? Explain your reasoning.

$w + \frac{7}{4} > \frac{3}{4}$ $\quad$ $w - \frac{3}{4} > -\frac{7}{4}$ $\quad$ $w + \frac{7}{4} < \frac{3}{4}$ $\quad$ $\frac{3}{4} < w + \frac{7}{4}$

Practice and Problem Solving

Solve the inequality. Graph the solution.

3. $x + 7 \geq 18$
4. $a - 2 > 4$
5. $3 \leq 7 + g$
6. $8 + k \leq -3$
7. $-12 < y - 6$
8. $n - 4 < 5$
9. $t - 5 \leq -7$
10. $p + \frac{1}{4} \geq 2$
11. $\frac{2}{7} > b + \frac{5}{7}$
12. $z - 4.7 \geq -1.6$
13. $-9.1 < d - 6.3$
14. $\frac{8}{5} > s + \frac{12}{5}$
15. $-\frac{7}{8} \geq m - \frac{13}{8}$
16. $r + 0.2 < -0.7$
17. $h - 6 \leq -8.4$

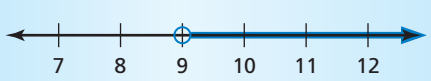
ERROR ANALYSIS **Describe and correct the error in solving the inequality or graphing the solution of the inequality.**

18.

$x - 7 > -2$

$+7 \quad +7$

$x > 9$

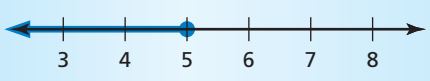


19.

$8 \leq x + 3$

$-3 \quad -3$

$5 \leq x$



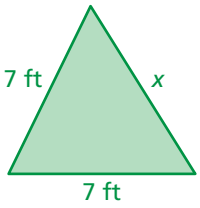
20. **AIRPLANE** A small airplane can hold 44 passengers. Fifteen passengers board the plane.

a. Write and solve an inequality that represents the additional number of passengers that can board the plane.

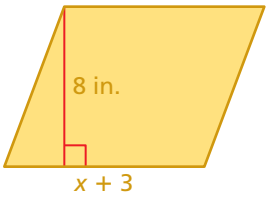
b. Can 30 more passengers board the plane? Explain.

Write and solve an inequality that represents x.

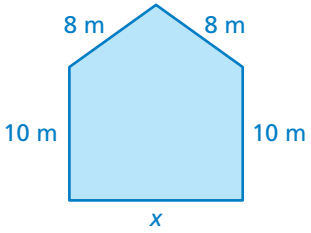
21. The perimeter is less than 28 feet.



22. The base is greater than the height.



23. The perimeter is less than or equal to 51 meters.



24. **REASONING** The solution of $d + s > -3$ is $d > -7$. What is the value of s?

25. **BIRDFEEDER** The hole for a birdfeeder post is 3 feet deep. The top of the post needs to be at least 5 feet above the ground. Write and solve an inequality that represents the required length of the post.

26. **SHOPPING** You can spend up to $35 on a shopping trip.

a. You want to buy a shirt that costs $14. Write and solve an inequality that represents the amount of money you will have left if you buy the shirt.

b. You notice that the shirt is on sale for 30% off. How does this change the inequality?

c. Do you have enough money to buy the shirt that is on sale and a pair of pants that costs $23? Explain.

27. **POWER** A circuit overloads at 2400 watts of electricity. A portable heater that uses 1050 watts of electricity is plugged into the circuit.

a. Write and solve an inequality that represents the additional number of watts you can plug in without overloading the circuit.

b. In addition to the portable heater, what two other items in the table can you plug in at the same time without overloading the circuit? Is there more than one possibility? Explain.

Item	Watts
Aquarium	200
Hair dryer	1200
Television	150
Vacuum cleaner	1100

28. Number Sense The possible values of x are given by $x + 8 \leq 6$. What is the greatest possible value of $7x$?

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 3.4)*

29. $4x = 36$ **30.** $\frac{w}{3} = -9$ **31.** $-2b = 44$ **32.** $60 = \frac{3}{4}h$

33. **MULTIPLE CHOICE** Which fraction is equivalent to –2.4? *(Section 2.1)*

(A) $-\frac{12}{5}$ (B) $-\frac{51}{25}$ (C) $-\frac{8}{5}$ (D) $-\frac{6}{25}$

4.3 Solving Inequalities Using Multiplication or Division

Essential Question How can you use multiplication or division to solve an inequality?

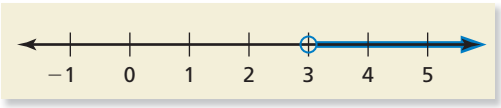
1 ACTIVITY: Using a Table to Solve an Inequality

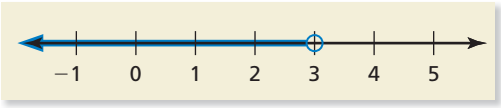
Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

a. $4x > 12$

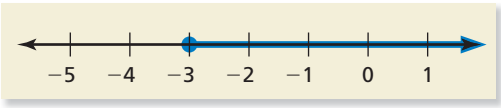
x	-1	0	1	2	3	4	5
$4x$							
$4x \stackrel{?}{>} 12$							

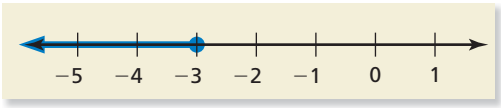




b. $-3x \leq 9$

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$-3x \stackrel{?}{\leq} 9$							





Common Core

Inequalities

In this lesson, you will

- solve inequalities using multiplication or division.
- solve real-life problems.

Learning Standard 7.EE.4b

2 ACTIVITY: Solving an Inequality

Work with a partner.

a. Solve $-3x \leq 9$ by adding $3x$ to each side of the inequality first. Then solve the resulting inequality.

b. Compare the solution in part (a) with the solution in Activity 1(b).

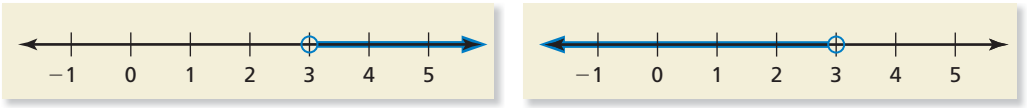
3 ACTIVITY: Using a Table to Solve an Inequality

Work with a partner.

- **Copy and complete the table.**
- **Decide which graph represents the solution of the inequality.**
- **Write the solution of the inequality.**

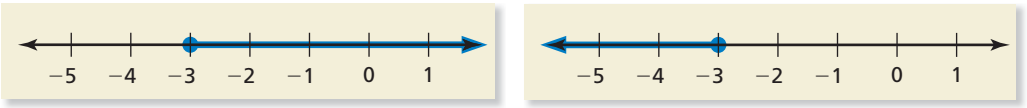
a. $\frac{x}{3} < 1$

x	-1	0	1	2	3	4	5
$\frac{x}{3}$							
$\frac{x}{3} \stackrel{?}{<} 1$							



b. $\frac{x}{-4} \geq \frac{3}{4}$

x	-5	-4	-3	-2	-1	0	1
$\frac{x}{-4}$							
$\frac{x}{-4} \stackrel{?}{\geq} \frac{3}{4}$							



4 ACTIVITY: Writing Rules

Work with a partner. Use a table to solve each inequality.

a. $-2x \leq 10$ **b.** $-6x > 0$ **c.** $\frac{x}{-4} < 1$ **d.** $\frac{x}{-8} \geq \frac{1}{8}$

Write a set of rules that describes how to solve inequalities like those in Activities 1 and 3. Then use your rules to solve each of the four inequalities above.

Math Practice 3

Analyze Conjectures

When you apply your rules to parts (a)–(d), do you get the same solutions? Explain.

What Is Your Answer?

5. IN YOUR OWN WORDS How can you use multiplication or division to solve an inequality?

Practice

Use what you learned about solving inequalities using multiplication or division to complete Exercises 4–9 on page 143.

4.3 Lesson

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Key Idea

Remember
Multiplication and division are inverse operations.

Multiplication and Division Properties of Inequality (Case 1)

Words When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Numbers

$-4 < 6$ $\qquad$ $4 > -6$

$2 \cdot (-4) < 2 \cdot 6$ $\qquad$ $\frac{4}{2} > \frac{-6}{2}$

$-8 < 12$ $\qquad$ $2 > -3$

Algebra If $a < b$ and c is positive, then

$a \cdot c < b \cdot c$ and $\frac{a}{c} < \frac{b}{c}$.

If $a > b$ and c is positive, then

$a \cdot c > b \cdot c$ and $\frac{a}{c} > \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 Solving an Inequality Using Multiplication

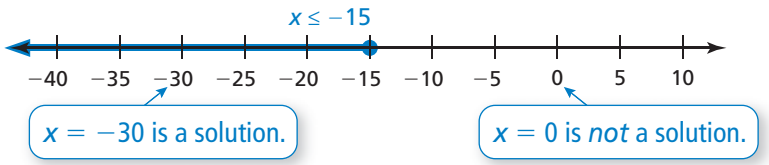
Solve $\frac{x}{5} \leq -3$. Graph the solution.

$\frac{x}{5} \leq -3$ Write the inequality.

Undo the division. → $5 \cdot \frac{x}{5} \leq 5 \cdot (-3)$ Multiplication Property of Inequality

$x \leq -15$ Simplify.

∴ The solution is $x \leq -15$.



On Your Own

Solve the inequality. Graph the solution.

1. $n \div 3 < 1$
2. $-0.5 \leq \frac{m}{10}$
3. $-3 > \frac{2}{3}p$

EXAMPLE 2 Solving an Inequality Using Division

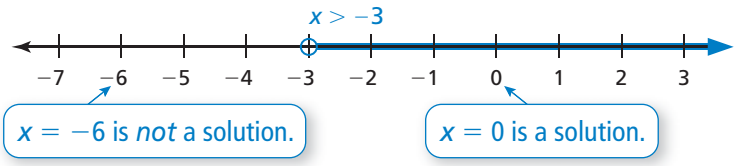
Solve $6x > -18$. Graph the solution.

$6x > -18$ Write the inequality.

Undo the multiplication. → $\frac{6x}{6} > \frac{-18}{6}$ Division Property of Inequality

$x > -3$ Simplify.

∴ The solution is $x > -3$.



On Your Own

Now You're Ready
Exercises 10–18

Solve the inequality. Graph the solution.

4. $4b \geq 2$ **5.** $12k \leq -24$ **6.** $-15 < 2.5q$

Key Idea

Multiplication and Division Properties of Inequality (Case 2)

Words When you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Numbers

$-4 < 6$ $\qquad$ $4 > -6$

$-2 \cdot (-4) > -2 \cdot 6$ $\qquad$ $\frac{4}{-2} < \frac{-6}{-2}$

$8 > -12$ $\qquad$ $-2 < 3$

Algebra If $a < b$ and c is negative, then

$a \cdot c > b \cdot c$ and $\frac{a}{c} > \frac{b}{c}$.

If $a > b$ and c is negative, then

$a \cdot c < b \cdot c$ and $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

Common Error

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol.

Only reverse the inequality symbol when you multiply or divide both sides by a negative number.

EXAMPLE 3 Solving an Inequality Using Multiplication

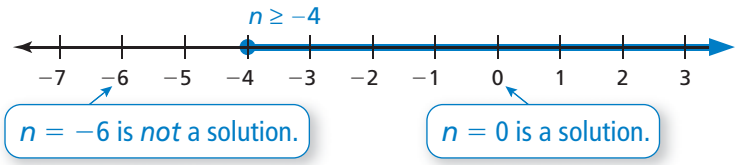
Solve $-\frac{3}{2}n \le 6$. Graph the solution.

$$-\frac{3}{2}n \le 6$$ Write the inequality.

$$-\frac{2}{3} \cdot \left(-\frac{3}{2}n\right) \ge -\frac{2}{3} \cdot 6$$ Use the Multiplication Property of Inequality. Reverse the inequality symbol.

$$n \ge -4$$ Simplify.

∴ The solution is $n \ge -4$.



On Your Own

Solve the inequality. Graph the solution.

7. $\frac{x}{-3} > -4$

8. $0.5 \le -\frac{y}{2}$

9. $-12 \ge \frac{6}{5}m$

10. $-\frac{2}{5}h \le -8$

EXAMPLE 4 Solving an Inequality Using Division

Solve $-3z > -4.5$. Graph the solution.

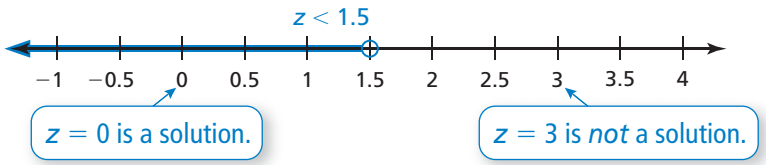
$$-3z > -4.5$$ Write the inequality.

Undo the multiplication. →

$$\frac{-3z}{-3} < \frac{-4.5}{-3}$$ Use the Division Property of Inequality. Reverse the inequality symbol.

$$z < 1.5$$ Simplify.

∴ The solution is $z < 1.5$.



On Your Own

Now You're Ready
Exercises 27–35

Solve the inequality. Graph the solution.

11. $-5z < 35$

12. $-2a > -9$

13. $-1.5 < 3n$

14. $-4.2 \ge -0.7w$

4.3 Exercises

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Vocabulary and Concept Check

1. **WRITING** Explain how to solve $\frac{x}{3} < -2$.
2. **PRECISION** Explain how solving $4x < -16$ is different from solving $-4x < 16$.
3. **OPEN-ENDED** Write an inequality that you can solve using the Division Property of Inequality where the direction of the inequality symbol must be reversed.

Practice and Problem Solving

Use a table to solve the inequality.

4. $2x < 2$
5. $-3x \leq 3$
6. $-6x > 18$
7. $\frac{x}{-5} \geq 7$
8. $\frac{x}{-1} > \frac{2}{5}$
9. $\frac{x}{3} \leq \frac{1}{2}$

Solve the inequality. Graph the solution.

10. $2n > 20$
11. $\frac{c}{9} \leq -4$
12. $2.2m < 11$
13. $-16 > x \div 2$
14. $\frac{1}{6}w \geq 2.5$
15. $7 < 3.5k$
16. $3x \leq -\frac{5}{4}$
17. $4.2y \leq -12.6$
18. $11.3 > \frac{b}{4.3}$

19. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

✗
$$\frac{x}{3} < -9$$
$$3 \cdot \frac{x}{3} > 3 \cdot (-9)$$
$$x > -27$$

Write the word sentence as an inequality. Then solve the inequality.

20. The quotient of a number and 4 is at most 5.
21. A number divided by 7 is less than -3.
22. Six times a number is at least -24.
23. The product of -2 and a number is greater than 30.
24. **SMART PHONE** You earn \$9.20 per hour at your summer job. Write and solve an inequality that represents the number of hours you need to work in order to buy a smart phone that costs \$299.

25. AVOCADOS You have $9.60 to buy avocados for a guacamole recipe. Avocados cost $2.40 each.

a. Write and solve an inequality that represents the number of avocados you can buy.

b. Are there infinitely many solutions in this context? Explain.

26. SCIENCE PROJECT Students in a science class are divided into 6 equal groups with at least 4 students in each group for a project. Write and solve an inequality that represents the number of students in the class.

Solve the inequality. Graph the solution.

27. $-5n \leq 15$

28. $-7w > 49$

29. $-\frac{1}{3}h \geq 8$

30. $-9 < -\frac{1}{5}x$

31. $-3y < -14$

32. $-2d \geq 26$

33. $4.5 > -\frac{m}{6}$

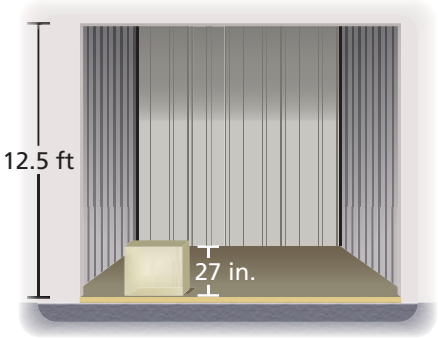
34. $\frac{k}{-0.25} \leq 36$

35. $-2.4 > \frac{b}{-2.5}$

36. ERROR ANALYSIS Describe and correct the error in solving the inequality.

$$-3m \geq 9$$
$$-\frac{3m}{-3} \geq \frac{9}{-3}$$
$$m \geq -3$$

37. TEMPERATURE It is currently 0°C outside. The temperature is dropping 2.5°C every hour. Write and solve an inequality that represents the number of hours that must pass for the temperature to drop below −20°C.



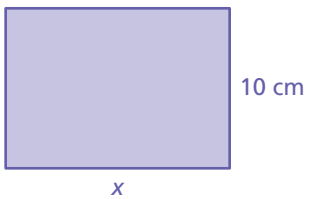
38. STORAGE You are moving some of your belongings into a storage facility.

a. Write and solve an inequality that represents the number of boxes that you can stack vertically in the storage unit.

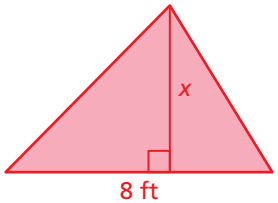
b. Can you stack 6 boxes vertically in the storage unit? Explain.

Write and solve an inequality that represents *x*.

39. Area ≥ 120 cm^2



40. Area < 20 ft^2



41. **AMUSEMENT PARK** You and four friends are planning a visit to an amusement park. You want to keep the cost below $100 per person. Write and solve an inequality that represents the total cost of visiting the amusement park.

42. **LOGIC** When you multiply or divide each side of an inequality by the same negative number, you must reverse the direction of the inequality symbol. Explain why.

43. **PROJECT** Choose two novels to research.

a. Use the Internet or a magazine to complete the table.

b. Use the table to find and compare the average number of copies sold per month for each novel. Which novel do you consider to be the most successful? Explain.

c. Assume each novel continues to sell at the average rate. Write and solve an inequality that represents the number of months it will take for the total number of copies sold to exceed twice the current number sold.

Author	Name of Novel	Release Date	Current Number of Copies Sold
1.			
2.			

Number Sense **Describe all numbers that satisfy *both* inequalities. Include a graph with your description.**

44. $4m > -4$ and $3m < 15$

45. $\frac{n}{3} \geq -4$ and $\frac{n}{-5} \geq 1$

46. $2x \geq -6$ and $2x \geq 6$

47. $-\frac{1}{2}s > -7$ and $\frac{1}{3}s < 12$

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 3.5)*

48. $-2w + 4 = -12$

49. $\frac{v}{5} - 6 = 3$

50. $3(x - 1) = 18$

51. $\frac{m + 200}{4} = 51$

52. **MULTIPLE CHOICE** What is the value of $\frac{2}{3} + \left(-\frac{5}{7}\right)$? *(Section 2.2)*

(A) $-\frac{3}{4}$ (B) $-\frac{1}{21}$ (C) $\frac{7}{10}$ (D) $1\frac{8}{21}$

4.4 Solving Two-Step Inequalities

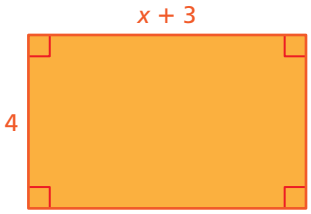
Essential Question How can you use an inequality to describe the dimensions of a figure?

1 ACTIVITY: Areas and Perimeters of Figures

Work with a partner.

- **Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.**
- **Write four values of the variable that satisfy the inequality you chose.**

a. You want to find the values of x so that the area of the rectangle is more than 22 square units.



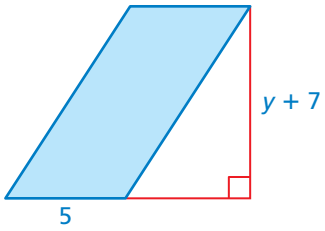
$4x + 12 > 22$ $4x + 3 > 22$

$4x + 12 \geq 22$ $2x + 14 > 22$

b. You want to find the values of x so that the perimeter of the rectangle is greater than or equal to 28 units.

$x + 7 \geq 28$ $4x + 12 \geq 28$ $2x + 14 \geq 28$ $2x + 14 \leq 28$

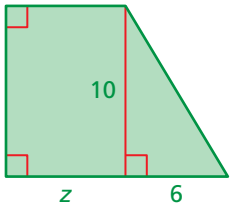
c. You want to find the values of y so that the area of the parallelogram is fewer than 41 square units.



$5y + 7 < 41$ $5y + 35 < 41$

$5y + 7 \leq 41$ $5y + 35 \leq 41$

d. You want to find the values of z so that the area of the trapezoid is at most 100 square units.



$5z + 30 \leq 100$ $10z + 30 \leq 100$

$5z + 30 < 100$ $10z + 30 < 100$

Common Core

Inequalities

In this lesson, you will

- solve multi-step inequalities.
- solve real-life problems.

Learning Standard 7.EE.4b

2 ACTIVITY: Volumes of Rectangular Prisms

Work with a partner.

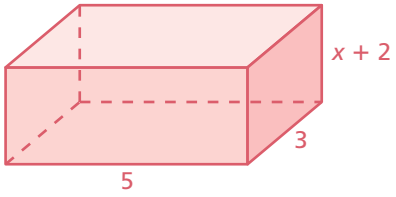
- **Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.**
- **Write four values of the variable that satisfy the inequality you chose.**

Math Practice 6

State the Meaning of Symbols

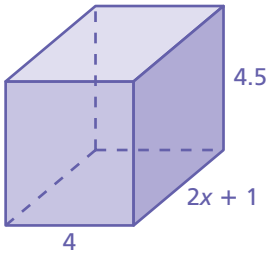
What inequality symbols do the phrases *at least* and *no more than* represent? Explain.

a. You want to find the values of x so that the volume of the rectangular prism is at least 50 cubic units.



$15x + 30 > 50$	$x + 10 \geq 50$	$15x + 30 \geq 50$	$15x + 2 \geq 50$

b. You want to find the values of x so that the volume of the rectangular prism is no more than 36 cubic units.



$8x + 4 < 36$	$36x + 18 < 36$	$2x + 9.5 \leq 36$	$36x + 18 \leq 36$

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use an inequality to describe the dimensions of a figure?
4. Use what you know about solving equations and inequalities to describe how you can solve a two-step inequality. Give an example to support your explanation.

Practice Use what you learned about solving two-step inequalities to complete Exercises 3 and 4 on page 150.

4.4 Lesson

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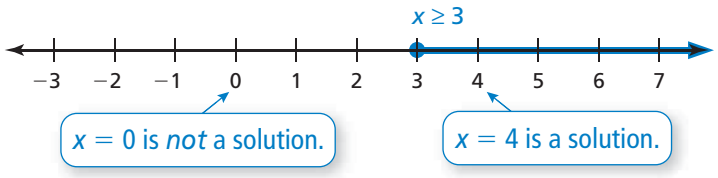
You can solve two-step inequalities in the same way you solve two-step equations.

EXAMPLE 1 Solving Two-Step Inequalities

a. Solve $5x - 4 \geq 11$. Graph the solution.

	$5x - 4 \geq 11$	Write the inequality.
Step 1: Undo the subtraction. →	$+4 \quad +4$	Addition Property of Inequality
	$5x \geq 15$	Simplify.
Step 2: Undo the multiplication. →	$\frac{5x}{5} \geq \frac{15}{5}$	Division Property of Inequality
	$x \geq 3$	Simplify.

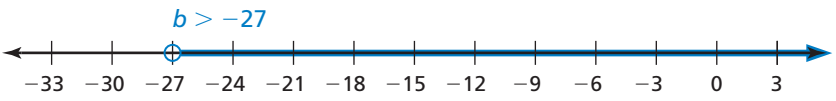
∴ The solution is $x \geq 3$.



b. Solve $\frac{b}{-3} + 4 < 13$. Graph the solution.

	$\frac{b}{-3} + 4 < 13$	Write the inequality.
Step 1: Undo the addition. →	$-4 \quad -4$	Subtraction Property of Inequality
	$\frac{b}{-3} < 9$	Simplify.
Step 2: Undo the division. →	$-3 \cdot \frac{b}{-3} > -3 \cdot 9$	Use the Multiplication Property of Inequality. Reverse the inequality symbol.
	$b > -27$	Simplify.

∴ The solution is $b > -27$.



On Your Own

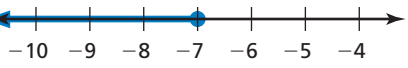
Now You're Ready
Exercises 5–10

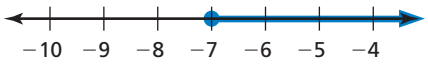
Solve the inequality. Graph the solution.

1. $6y - 7 > 5$
2. $4 - 3d \geq 19$
3. $\frac{w}{-4} + 8 > 9$

EXAMPLE 2 Graphing an Inequality

Which graph represents the solution of $-7(x + 3) \leq 28$?

Ⓐ 

Ⓑ 

Ⓒ

Ⓓ

$-7(x + 3) \leq 28$ — Write the inequality.

$-7x - 21 \leq 28$ — Distributive Property

Step 1: Undo the subtraction. → $+21 \quad +21$ — Addition Property of Inequality

$-7x \leq 49$ — Simplify.

Step 2: Undo the multiplication. → $\frac{-7x}{-7} \geq \frac{49}{-7}$ — Use the Division Property of Inequality. Reverse the inequality symbol.

$x \geq -7$ — Simplify.

∴ The correct answer is Ⓑ.

EXAMPLE 3 Real-Life Application

Progress Report	
Month	**Pounds Lost**
1	12
2	9
3	5
4	8

A contestant in a weight-loss competition wants to lose an average of at least 8 pounds per month during a 5-month period. How many pounds must the contestant lose in the fifth month to meet the goal?

Write and solve an inequality. Let x be the number of pounds lost in the fifth month.

$\frac{12 + 9 + 5 + 8 + x}{5} \geq 8$ — The phrase *at least* means *greater than or equal to.*

$\frac{34 + x}{5} \geq 8$ — Simplify.

$5 \cdot \frac{34 + x}{5} \geq 5 \cdot 8$ — Multiplication Property of Inequality

$34 + x \geq 40$ — Simplify.

$x \geq 6$ — Subtract 34 from each side.

∴ So, the contestant must lose at least 6 pounds to meet the goal.

Remember

In Example 3, the average is equal to the sum of the pounds lost divided by the number of months.

On Your Own

Now You're Ready
Exercises 12–17

Solve the inequality. Graph the solution.

4. $2(k - 5) < 6$ **5.** $-4(n - 10) < 32$ **6.** $-3 \leq 0.5(8 + y)$

7. WHAT IF? In Example 3, the contestant wants to lose an average of at least 9 pounds per month. How many pounds must the contestant lose in the fifth month to meet the goal?

4.4 Exercises

Check It Out
Help with Homework
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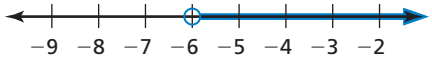
Vocabulary and Concept Check

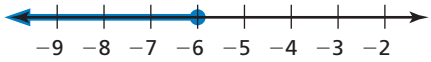
1. **WRITING** Compare and contrast solving two-step inequalities and solving two-step equations.
2. **OPEN-ENDED** Describe how to solve the inequality $3(a + 5) < 9$.

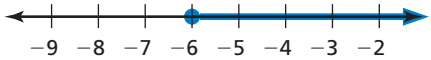
Practice and Problem Solving

Match the inequality with its graph.

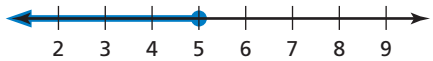
3. $\frac{t}{3} - 1 \geq -3$

A. 

B. 

C. 

4. $5x + 7 \leq 32$

A. 

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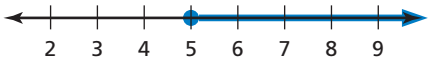
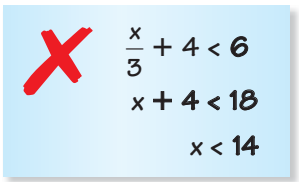
C. 

Solve the inequality. Graph the solution.

1 **5.** $8y - 5 < 3$ **6.** $3p + 2 \geq -10$ **7.** $2 > 8 - \frac{4}{3}h$

8. $-2 > \frac{m}{6} - 7$ **9.** $-1.2b - 5.3 \geq 1.9$ **10.** $-1.3 \geq 2.9 - 0.6r$

11. ERROR ANALYSIS Describe and correct the error in solving the inequality.



Solve the inequality. Graph the solution.

2 **12.** $5(g + 4) > 15$ **13.** $4(w - 6) \leq -12$ **14.** $-8 \leq \frac{2}{5}(k - 2)$

15. $-\frac{1}{4}(d + 1) < 2$ **16.** $7.2 > 0.9(n + 8.6)$ **17.** $20 \geq -3.2(c - 4.3)$

18. UNICYCLE The first jump in a unicycle high-jump contest is shown. The bar is raised 2 centimeters after each jump. Solve the inequality $2n + 10 \geq 26$ to find the number of additional jumps needed to meet or exceed the goal of clearing a height of 26 centimeters.

Solve the inequality. Graph the solution.

19. $9x - 4x + 4 \geq 36 - 12$

20. $3d - 7d + 2.8 < 5.8 - 27$

21. **SCUBA DIVER** A scuba diver is at an elevation of -38 feet. The diver starts moving at a rate of -12 feet per minute. Write and solve an inequality that represents how long it will take the diver to reach an elevation deeper than -200 feet.

22. **KILLER WHALES** A killer whale has eaten 75 pounds of fish today. It needs to eat at least 140 pounds of fish each day.

a. A bucket holds 15 pounds of fish. Write and solve an inequality that represents how many more buckets of fish the whale needs to eat.

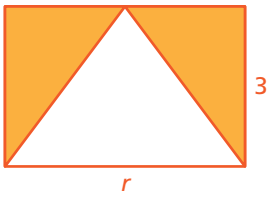
b. Should the whale eat *four* or *five* more buckets of fish? Explain.

23. **REASONING** A student theater charges \$9.50 per ticket.

a. The theater has already sold 70 tickets. Write and solve an inequality that represents how many more tickets the theater needs to sell to earn at least \$1000.

b. The theater increases the ticket price by \$1. Without solving an inequality, describe how this affects the total number of tickets needed to earn at least \$1000.

24. Problem Solving For what values of r will the area of the shaded region be greater than or equal to 12 square units?



Fair Game Review What you learned in previous grades & lessons

Find the missing values in the ratio table. Then write the equivalent ratios. *(Skills Review Handbook)*

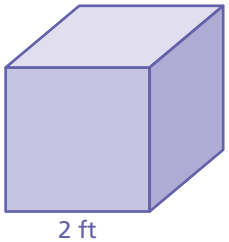
25.

Flutes	7		28
Clarinets	4	12	

26.

Boys	6	3	
Girls	10		50

27. **MULTIPLE CHOICE** What is the volume of the cube? *(Skills Review Handbook)*



Ⓐ $8\ ft^3$ Ⓑ $16\ ft^3$

Ⓒ $24\ ft^3$ Ⓓ $32\ ft^3$